BER Analysis of OFDM Systems Impaired by Phase Noise in Frequency-Selective Rayleigh Fading Channels

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Abstract— **In this paper, we study the effect of finite-power, phase-locked-loop (PLL) based phase noise on the bit-error-rate (BER) of orthogonal frequency division multiplexing (OFDM) systems in frequency-selective Rayleigh fading channels. Based on the conditional Gaussian approximation technique, we derive the BER formulas for BPSK and 16-QAM modulated OFDM signals impaired by phase noise in frequency-selective Rayleigh fading channels. Simulation results not only validate the accuracy of our analysis but also show the dependency of BERs on the shapes of phase noise spectra.**

I. Introduction

The effects of phase noise and its mitigation methods for OFDM systems have been extensively researched and reported in literature [1]-[13]. In [1], the two effects of phase noise on OFDM systems, namely, the common phase error (CPE) and ICI were analyzed by deriving their second-order statistics. The authors also proposed a simple CPE correction technique based on the continual pilot signals. The dependency of ICI and CPE powers on the power spectrum density of the phase noise was further characterized in [2]. The signal-to-noise ratio (SNR) degradation of OFDM systems caused by the presence of phase noise in additive white Gaussian channels (AWGN) and multipath Rayleigh fading channels were evaluated in [3]-[6]. Although SNR is a simple way to assess the impact of phase noise on OFDM systems, BER is a more important and accurate performance metric. Based on the moment generating function (MGF) technique and the Gaussian approximation of ICI, the BER analysis of OFDM systems impaired by Wiener phase noise in AWGN channels was presented in [7]. For the case of finite-power phase noises, the BER results can be found in [8]-[11]. However, it has been indicated in [12] and later analyzed in [13], the Gaussian approximation of ICI is accurate only when the phase noise has larger bandwidth than the OFDM subcarrier spacing (i.e. fast phase noise) and the number of subcarriers is large.

Since OFDM systems are usually designed to counter the detrimental effects of frequency-selective fading channels, it is important to analytically quantify the BER performance in the presence of both finite-power phase noise and frequency-selective fading. This aspect has not been considered in the aforementioned works. We apply the BER analysis technique developed in [14] to the phase noise problem and derive the BER formulas for various modulations without and with perfect CPE compensation. Since the channel frequency responses and the phase noise frequency responses of different subcarriers are correlated, the signal term and the ICI term for the data symbol transmitted at each subcarrier are correlated. Instead of directly invoking the central limit theorem (CLT) [15] to approximate the ICI as a Gaussian random variable, we first condition the received signal on the channel realization and CPE. Then we approximate the ICI plus noise term as a Gaussian random variable and derive the conditional BER expressions. Finally, the conditional BER formulas are averaged over the conditional random variables to obtain the average BERs. Numerical results show that the conditional Gaussian approximation technique yields accurate BER predictions for various modulation formats and phase noise spectra.

The notations $(\cdot)^{\mathsf{T}}$ and $(\cdot)^*$ stand for matrix transpose and complex conjugation, respectively. The expectation taken with respect to the distribution of the random variable within the brackets is denoted by $\mathbb{E}\{\cdot\}$. Finally, the symbol $\mathcal{CN}(0,\sigma^2)$ denotes the circularly symmetric complex Gaussian random variable with mean 0 and variance $\sigma^2/2$ in both real and imaginary components.

II. System Model

A. Received Signal

The transmitted discrete-time OFDM baseband signal in one OFDM symbol period is given by

$$x_i = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k\, e^{j2\pi ik/N},\ i=0,1,\cdots,N-1 \quad (1)$$

where k is the index of subcarrier, N is the total number of subcarriers, and $X_k \in \mathcal{X}$ is the transmitted modulation symbol at subcarrier k. For different subcarrier index k, X_k are independent with average symbol energy $E_s = \mathbb{E}[|X_k|^2]$. Two types of constellations $\mathcal{X}$ are considered in this paper, namely, BPSK and 16-QAM.

At the transmitter, we assume the cyclic prefix whose length is longer than the maximum delay spread of the multipath fading channel is inserted at the beginning of each OFDM symbol. At the receiver, the inserted cyclic prefix is discarded in the demodulation process. Therefore, no intersymbol interference (ISI) occurs in the received OFDM signals.

The transmitted OFDM signal goes through a time-invariant frequency-selective Rayleigh fading channel whose impulse response is represented by the tapped-delay line model as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - lT/N), \quad (2)$$

where L is the number of multipaths, the path gain $h_l \sim \mathcal{CN}(0,\sigma_l^2)$, $\delta(\cdot)$ is the Dirac-delta function, and T is the

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effective OFDM symbol period. The corresponding frequency response at subcarrier i is

$$H_i = \int_{-\infty}^{\infty} h(t)e^{-j2\pi it/T}dt = \sum_{l=0}^{L-1} h_l e^{-j2\pi il/N}. \quad (3)$$

Since h_l are complex Gaussian random variables, H_i are also complex Gaussian random variables with mean 0 and variance $\sum_{l=0}^{L-1}\sigma_l^2$. Without loss of generality, we assume the sum of the average power of each multipath is normalized to 1, i.e. $\sum_{l=0}^{L-1}\sigma_l^2 = 1$.

The received discrete-time OFDM baseband signal impaired by phase noise and AWGN is given by

$$y_i = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} H_k X_k e^{j(2\pi ik/N+\psi_i)} + w_i, i = 0, 1, \cdots, N-1, \quad (4)$$

where ψ_i are the sampled phase noises and w_i are identically independently distributed (i.i.d.) from $\mathcal{CN}(0, N_0)$. Taking the fast Fourier transform (FFT) of the received samples $y_i, i = 0, 1, \cdots, N-1$, we have

$$Y_n = \frac{1}{N}\sum_{i=0}^{N-1} e^{j\psi_i}\sum_{k=0}^{N-1} H_k X_k e^{j2\pi(k-n)i/N} + W_n, \quad (5)$$

where $W_n = \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1} w_i\, e^{-j2\pi ni/N},\ n = 0, 1, \cdots, N-1$ are i.i.d. from $\mathcal{CN}(0, N_0)$. In most practical OFDM systems, the phase noise ψ_i is small (i.e., $|\psi_i| << 1$) so that we can use the approximation $e^{j\psi_i} \approx 1 + j\psi_i$ to simplify our analysis. In this case, Y_n can be expressed as

$$Y_n \approx \frac{1}{N}\sum_{i=0}^{N-1}(1 + j\psi_i)\sum_{k=0}^{N-1} H_k X_k e^{j2\pi(k-n)i/N} + W_n$$
$$= H_n X_n + j\sum_{k=0}^{N-1} H_k X_k \Psi_{k-n} + W_n,\ n = 0, \cdots, N-1 \quad (6)$$

where $\Psi_k = \frac{1}{N}\sum_{i=0}^{N-1}\psi_i\, e^{j2\pi ki/N}$ is the frequency response of phase noise samples ψ_i at subcarrier k. The second term on the right-hand side (RHS) of (6) consists of

$$j\sum_{k=0}^{N-1} H_k X_k \Psi_{k-n} = jX_n H_n \Psi_0 + j\sum_{k=0,k\neq n}^{N-1} H_k X_k \Psi_{k-n}, \quad (7)$$

where $jH_n X_n \Psi_0$ is the common phase error (CPE) because Ψ_0 is the same for all subcarriers. The term $j\sum_{k=0,k\neq n}^{N-1} H_k X_k \Psi_{k-n}$ is the ICI resulting from the loss of orthogonality between different subcarriers.

B. Phase Noise Model

We focus on the phase noise in the local oscillator synthesized by PLL. Let $\psi(t)$ be the continuous phase noise process which is assumed to be a stationary Gaussian random process with zero mean and PSD specified by a phase noise mask $\Omega(f)$. For the purpose of performance analysis in next section, we are interested in the second-order statistical property of the discrete-time phase noise samples ψ_i. Let f_s be the sampling frequency and $T_s = 1/f_s$ be the sampling period. The ith phase noise sample is denoted by $\psi_i = \psi(iT_s)$. Then the autocorrelation function of the phase noise samples ψ_i is related to the continuous phase noise spectrum $\Omega(f)$ by [9]

$$R(k) \triangleq \mathbb{E}[\psi_i\psi_{i+k}] = \int_{-\infty}^{\infty}\Omega(f)e^{-j2\pi kf/f_s}df, \quad (8)$$

and the phase noise power σ_ψ^2 is equal to $R(0)$.

Two specific phase noise spectra [12] are considered in this paper. The first phase noise spectrum, referred as "spectrum A", can be described mathematically as [1]

$$\Omega_A(f) = 10^{-c} + \begin{cases} 10^{-a}, & |f| \le f_1 \\ 10^{-\frac{b(f-f_1)}{f_2-f_1}-a}, & f_1 < f < \frac{f_s}{2} \\ 10^{-\frac{b(f+f_1)}{f_2-f_1}-a}, & -\frac{f_s}{2} < f < -f_1 \end{cases}. \quad (9)$$

The second phase noise spectrum, referred as "spectrum B", is given by

$$\Omega_B(f) = \frac{1}{\pi}\frac{P\beta}{f^2+\beta^2},\ -\frac{f_s}{2} < f < \frac{f_s}{2} \quad (10)$$

where β is the phase noise 3-dB bandwidth and P is the total phase noise power. A plot of $\Omega_A(f)$ and $\Omega_B(f)$ with typical parameters is shown in Fig. 1.

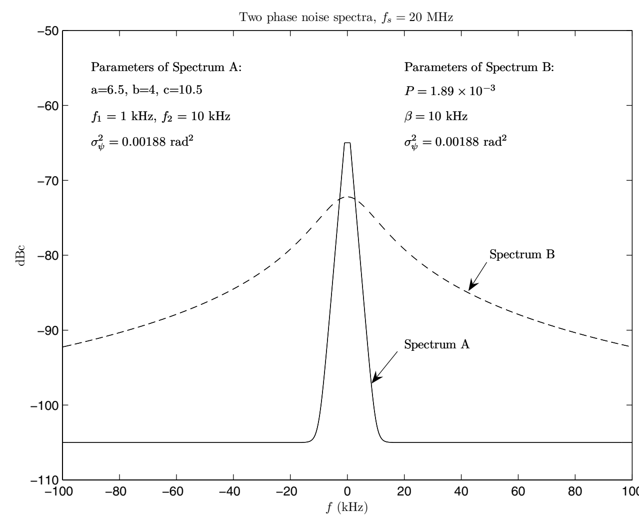

Fig. 1. Two phase noise spectra of phase-locked oscillators.

III. BER Analysis

To derive the BER formula, without loss of generality, we consider subcarrier 0 as the desired subcarrier and the received OFDM signal in (6) can be rewritten as

$$Y_0 = H_0 X_0 + jH_0 X_0 \Psi_0 + V_0, \quad (11)$$

where $V_0 = j\sum_{k=1}^{N-1} H_k X_k \Psi_k + W_0$ is the sum of ICI and AWGN. Since V_0 is the sum of N uncorrelated random variables $jH_k X_k \Psi_k$ and W_0, one may try to invoke the CLT to approximate V_0 as a Gaussian random variable with zero mean and suitable variance when N is large. However, for the AWGN channel, it has been shown in [13] that approximating

V_0 as a Gaussian random variable is not valid and the ICI term has a thicker tail. When frequency-selective fading is present, directly applying the CLT to V_0 may overestimate the BER since the channel response H_0 of the desired signal is correlated with the channel responses $H_1, H_2, \cdots, H_{N-1}$ of the interference term V_0. That means the signal power and the interference power are correlated. Moreover, we note that the phase noise response Ψ_0 is also strongly correlated with $\Psi_1, \Psi_2, \cdots, \Psi_{N-1}$.

To compute the BER accurately, we consider to adapt the performance analysis method developed in [14] to solve our problems. First, we define the conditional random variables $\xi_k = H_k|H_0$ and $\zeta_k = \Psi_k|\Psi_0$ where $k = 1, 2, \cdots, N-1$. Since H_0 and H_k are jointly complex Gaussian, the conditional random variable ξ_k are also complex Gaussian random variables [15] with mean and variance

$$\mathbb{E}[\xi_k] = \mathbb{E}[H_k|H_0] = H_0\frac{\rho_{k0}}{\rho_{00}}, \quad \mathrm{Var}[\xi_k] = \rho_{kk} - \frac{|\rho_{k0}|^2}{\rho_{00}}, \tag{12}$$

where

$$\rho_{ik} = \mathbb{E}[H_i H_k^*] = \sum_{l=0}^{L-1} \sigma_l^2 e^{j2\pi(k-i)l/N}. \tag{13}$$

Similarly, since Ψ_k and Ψ_0 are jointly complex Gaussian, the conditional random variable ζ_k are also complex Gaussian random variables with mean and variance

$$\mathbb{E}[\zeta_k] = \mathbb{E}[\Psi_k|\Psi_0] = \Psi_0\frac{\hat{\rho}_{k0}}{\hat{\rho}_{00}}, \quad \mathrm{Var}[\zeta_k] = \hat{\rho}_{kk} - \frac{|\hat{\rho}_{k0}|^2}{\hat{\rho}_{00}}, \tag{14}$$

where

$$\hat{\rho}_{ik} = \mathbb{E}[\Psi_i \Psi_k^*] = \frac{1}{N^2}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} e^{j2\pi(in_1-kn_2)/N} R(n_1-n_2). \tag{15}$$

To facilitate further performance analysis, we define two zero-mean random variables

$$\alpha_k = \xi_k - \mathbb{E}[\xi_k] \sim \mathcal{CN}(0, \rho_{kk} - \rho_{00}^{-1}|\rho_{k0}|^2), \tag{16}$$

$$\beta_k = \zeta_k - \mathbb{E}[\zeta_k] \sim \mathcal{CN}(0, \hat{\rho}_{kk} - \hat{\rho}_{00}^{-1}|\hat{\rho}_{k0}|^2). \tag{17}$$

Conditioned on H_0 and Ψ_0, the received signal Y_0 becomes

$$\begin{aligned} Z_0 \triangleq Y_0|H_0, \Psi_0 &= H_0X_0 + jH_0X_0\Psi_0 + j\sum_{k=1}^{N-1}\xi_k\zeta_k X_k + W_0 \\ &= H_0X_0 + jH_0X_0\Psi_0 + I_0, \end{aligned} \tag{18}$$

where $I_0 = j\sum_{k=1}^{N-1}\left(\alpha_k + H_0\rho_{00}^{-1}\rho_{k0}\right)\left(\beta_k + \Psi_0\hat{\rho}_{00}^{-1}\hat{\rho}_{k0}\right)X_k + W_0$. When the number of subcarriers N is large, we can invoke the CLT to approximate I_0 as a complex Gaussian random variable with mean zero and variance

$$\begin{aligned} &\mathrm{Var}[|I_0|^2] \triangleq E_s\sigma_{I_0}^2(|H_0|^2, \Psi_0) \\ &= E_s\left[\lambda_1 + \lambda_2(\Psi_0) + \lambda_3(|H_0|^2) + \lambda_4(|H_0|^2, \Psi_0) + \lambda_5\right], \end{aligned} \tag{19}$$

where

$$\lambda_1 = \sum_{k=1}^{N-1}\left(\rho_{kk} - \rho_{00}^{-1}|\rho_{k0}|^2\right)\left(\hat{\rho}_{kk} - \hat{\rho}_{00}^{-1}|\hat{\rho}_{k0}|^2\right), \tag{20}$$

$$\lambda_2(\Psi_0) = \sum_{k=1}^{N-1}\left(\rho_{kk} - \rho_{00}^{-1}|\rho_{k0}|^2\right)\left|\hat{\rho}_{00}^{-1}\hat{\rho}_{k0}\right|^2\Psi_0^2, \tag{21}$$

$$\lambda_3(|H_0|^2) = \sum_{k=1}^{N-1}\left(\hat{\rho}_{kk} - \hat{\rho}_{00}^{-1}|\hat{\rho}_{k0}|^2\right)\left|\rho_{00}^{-1}\rho_{k0}\right|^2|H_0|^2, \tag{22}$$

$$\lambda_4(|H_0|^2, \Psi_0) = \sum_{k=1}^{N-1}\left|\rho_{00}^{-1}\rho_{k0}\hat{\rho}_{00}^{-1}\hat{\rho}_{k0}\right|^2|H_0|^2\Psi_0^2, \tag{23}$$

$$\lambda_5 = N_0/E_s. \tag{24}$$

In (19), we explicitly express the variance of I_0 as a function of $|H_0|^2$ and Ψ_0. Next, we will derive the BER formulas for various modulation formats under the assumption that perfect channel state information (CSI) is available at the receiver.

A. BPSK

The constellation $\mathcal{X}$ of BPSK modulation is $\{-\sqrt{E_s}, \sqrt{E_s}\}$. Conditioned on H_0 and Ψ_0, the conditional BER is given by

$$\begin{aligned} P_b(e|H_0, \Psi_0) &= \Pr\left(\mathrm{Re}[Z_0H_0^*] < 0|X_0 = \sqrt{E_s}\right) \\ &= \Pr\left(|H_0|^2\sqrt{E_s} + \mathrm{Re}[I_0H_0^*] < 0\right) \\ &= Q\left(\sqrt{\frac{2|H_0|^2}{\sigma_{I_0}^2(|H_0|^2, \Psi_0)}}\right), \end{aligned} \tag{25}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-z^2/2}dz$. From (25), we know the CPE term $jH_0X_0\Psi_0$ does not affect the BER performance of BPSK modulation. Since $H_0 \sim \mathcal{CN}(0,1)$, the probability density function (pdf) of $|H_0|^2$ is

$$f_{|H_0|^2}(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}. \tag{26}$$

On the other hand, the random variable $\Psi_0 = \frac{1}{N}\sum_{i=0}^{N-1}\psi_i$ is the sum of N zero mean Gaussian random variables. Therefore, Ψ_0 is also a zero mean Gaussian random variable with variance $\sigma_{\Psi_0}^2 = \hat{\rho}_{00}$, which has been given explicitly in (15). The pdf of Ψ_0 is given by

$$f_{\Psi_0}(y) = \frac{1}{\sqrt{2\pi\sigma_{\Psi_0}^2}}e^{-y^2/2\sigma_{\Psi_0}^2}, \quad -\infty < y < \infty. \tag{27}$$

Averaging the conditional BER (25) over the pdfs of $|H_0|^2$ and Ψ_0, the unconditional BER can be expressed as

$$P_b(e) = \int_{-\infty}^{\infty}\int_0^{\infty} Q\left(\sqrt{\frac{2x}{\sigma_{I_0}^2(x,y)}}\right) f_{|H_0|^2}(x) f_{\Psi_0}(y)\,dx\,dy. \tag{28}$$

By suitable change of variables, the double integral in (28) can be evaluated accurately by using the Gauss-Chebyshev quadrature rule [16].

B. 16-QAM

Let $\mathcal{X} = \left\{\frac{[(2i-3)+(2q-3)j]\sqrt{E_s}}{\sqrt{10}}, \ i = 0,1,2,3; \ q = 0,1,2,3\right\}$ denote the 16-QAM constellation shown in Fig. 2(a). Each constellation point has equal *a priori* probability. The first and

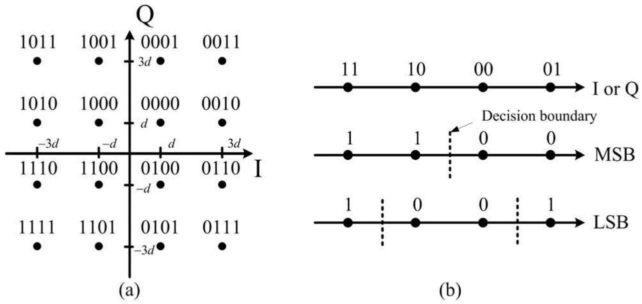

Fig. 2. (a) 16-QAM constellation with Gray encoding. (b) 16-QAM bit-by-bit demapping.

third bits correspond to the inphase (I) bits, while the second and fourth bits map to the quadrature (Q) bits. The I and Q components of the 16-QAM symbols are Gray encoded by assigning the bits 11, 10, 00, 01 to the levels $-3d$, $-d$, d, and $3d$ where $d = \sqrt{E_s/10}$. The decision boundaries for the most significant bit (MSB) and least significant bit (LSB) of the I/Q components are shown in Fig. 2(b). Due to the symmetry of I and Q components, the BERs of the I bits and the Q bits are the same. For the purpose of BER analysis, we only need to compute the BER of the I bits.

We define two sets $\mathcal{X}_1 = \{x \in \mathcal{X} : \mathrm{Re}[x] = d\}$ and $\mathcal{X}_2 = \{x \in \mathcal{X} : \mathrm{Re}[x] = 3d\}$. Since the decision boundary for the MSB of I bits is the imaginary axis, the BER of the MSB of I bits conditioned on H_0 and Ψ_0 is

$$P_b^{\mathrm{MSB}}(e|H_0, \Psi_0) = \frac{1}{8} \sum_{X_0 \in \mathcal{X}_1 \cup \mathcal{X}_2} \Pr\left(\mathrm{Re}[Z_0 H_0^*] < 0 | X_0\right) = \frac{1}{8} \sum_{X_0 \in \mathcal{X}_1 \cup \mathcal{X}_2} Q\left(\sqrt{\frac{2|H_0|^2}{E_s \sigma_{I_0}^2(|H_0|^2, \Psi_0)}}\ (\mathrm{Re}[X_0] - \mathrm{Im}[X_0]\Psi_0)\right). \quad (29)$$

On the other hand, the decision boundaries for the LSB are $I = -2d$ and $I = 2d$ on the I-Q plane. Therefore, the BER of the LSB of the I bits conditioned on H_0 and Ψ_0 is

$$\begin{aligned} P_b^{\mathrm{LSB}}(e|H_0, \Psi_0) = & \\ \frac{1}{8}\Bigg\{ & \sum_{X_0 \in \mathcal{X}_1} \left[1 - \Pr\left(-2d < \frac{\mathrm{Re}[Z_0 H_0^*]}{|H_0|^2} < 2d | X_0\right)\right] \\ & + \sum_{X_0 \in \mathcal{X}_2} \Pr\left(-2d < \frac{\mathrm{Re}[Z_0 H_0^*]}{|H_0|^2} < 2d | X_0\right)\Bigg\} \\ = & \ \frac{1}{8} \sum_{X_0 \in \mathcal{X}_1} [1 - g(-2d, X_0, H_0, \Psi_0) + g(2d, X_0, H_0, \Psi_0)] \\ + & \ \frac{1}{8} \sum_{X_0 \in \mathcal{X}_2} [g(-2d, X_0, H_0, \Psi_0) - g(2d, X_0, H_0, \Psi_0)], \quad (30) \end{aligned}$$

where $g(u, X_0.H_0, \Psi_0) =$

$$Q\left(\sqrt{\frac{2|H_0|^2}{E_s \sigma_{I_0}^2(|H_0|^2, \Psi_0)}}\ (u - \mathrm{Re}[X_0] + \mathrm{Im}[X_0]\Psi_0)\right).$$

The average conditional BER of 16-QAM modulated OFDM signals is

$$P_b^{16\mathrm{qam}}(e|H_0, \Psi_0) = \frac{\left[P_b^{\mathrm{MSB}}(e|H_0, \Psi_0) + P_b^{\mathrm{LSB}}(e|H_0, \Psi_0)\right]}{2}. \quad (31)$$

Finally, the unconditional average BER $P_b^{16\mathrm{qam}}(e)$ can be obtained by averaging the conditional probability $P_b(e|H_0, \Psi_0)$ over the pdfs $f_{|H_0|^2}(x)$ and $f_{\Psi_0}(y)$. Since the pdf $f_{\Psi_0}(y)$ is symmetric with respect to the origin, the computation of $P_b^{16\mathrm{qam}}(e)$ can be further simplified. We can replace $\mathcal{X}_1$, $\mathcal{X}_2$, and the constant $1/8$ in (29) and (30) by $\widetilde{\mathcal{X}}_1 = \{x \in \mathcal{X} : \mathrm{Re}[x] = d,\ \mathrm{Im}[x] > 0\}$, $\widetilde{\mathcal{X}}_2 = \{x \in \mathcal{X} : \mathrm{Re}[x] = 3d,\ \mathrm{Im}[x] > 0\}$ and 1/4, respectively.

V. Numerical Results

A. Simulation Setup

We consider an OFDM system with $N = 64$ subcarriers. The effective OFDM symbol period is $T = 3.2$ μs and the subcarrier frequency spacing Δf is 312.5 KHz. The sampling rate of the received signal is $f_s = 20$ MHz. The power delay profile of the frequency-selective Rayleigh fading channel is exponentially decaying and the root mean square (rms) delay spread is equal to 100 ns. We also assume the channel is fixed for the whole frame and is independent from frame to frame. These parameters and assumptions are typical for the indoor wireless local area network (WLAN) applications. As for the parameters in the mathematical models of the phase noise spectra, we set $a = 6.5$, $b = 4$, $c = 10.5$, $f_1 = 1$ kHz, and $f_2 = 10$ kHz for phase noise spectrum A. The parameter β and P in the phase noise spectrum B are set to be 10 kHz and 0.001, respectively [12]. To study the effect of different phase noise powers σ_ψ^2 on the BER performance, we simply scale the two phase noise spectra by multiplying suitable constants.

For each of the phase noise spectra considered in this paper, the ratio of the 3-dB bandwidth and the subcarrier spacing is much smaller than 1. This corresponds to the practical case of slow phase noise and the BERs computed based on the conventional Gaussian approximation are very inaccurate [13]. In next subsection, we will demonstrate the BERs can be accurately predicted by using the conditional Gaussian approximation technique.

B. Results

Fig. 3 and Fig. 4 show the effect of phase noise on the BER performance of 16-QAM modulated OFDM signals in frequency-selective Rayleigh fading channels with phase noise spectrum A and B, respectively. The horizontal axis represents the modulated data symbol SNR E_s/N_0 and the vertical axis represents the BER. The dashed lines are obtained from exact computer simulation without any approximation. On the other hand, the solid lines are obtained from computer simulation with approximation $e^{j\psi_i} \approx 1 + j\psi_i$ when $|\psi_i| << 1$. The markers are computed from our theoretical analysis. From Figs. 3- 4, we can see the theoretical results and the simulation results with approximation are matched very well. However, the exact BER is slightly higher than the approximate BER especially when the phase noise power σ_ψ^2 is as large as 0.01. Since we use the approximation $e^{j\psi_i} \approx 1 + j\psi_i$ to simplify our analysis, it is worth mentioning that our analysis is accurate only when $|\psi_i|$ is far smaller than 1. Fortunately, this condition holds in most practical OFDM systems.

By comparing Fig. 3 and Fig. 4, we learn the BER performance of phase noise spectrum A is much worse than that of phase noise spectrum B when they both have the same phase noise power σ_ψ^2. Since the CPE term $\Psi_0 = \frac{1}{N}\sum_{i=0}^{N-1}\psi_i$ is a zero mean Gaussian random variable, we know the variance of Ψ_0 depends not only on the variance of ψ_i but also on the correlation between any two different phase noise samples. Different phase noise spectra may have different autocorrelation functions $R(k)$ (see (8)), hence different CPE variances. Besides, the shape of the phase noise spectrum also affects the ICI variance through $\hat{\rho}_{ik}$ whose explicit form (13) is also a function of $R(k)$.

VI. Conclusions

This paper studied the effect of phase noise on the BER performance of OFDM systems in frequency-selective Rayleigh fading channels. For BPSK and 16-QAM modulated OFDM signals, we derived the BER formulas characterizing the performance degradation due to phase noise without CPE correction based on the conditional Gaussian approximation. Numerical results demonstrate the accuracy of our analysis. Depending on the shapes of phase noise spectra, the phase noise effect can be dominated by CPE or ICI, or both. While the forward CPE compensation has been applied in many practical OFDM systems, the ICI suppression and/or cancellation schemes usually have high computational complexity to hinder their implementation. The analysis and design of low-complexity ICI compensation methods to counter the detrimental effect of phase noise are left for further investigation.

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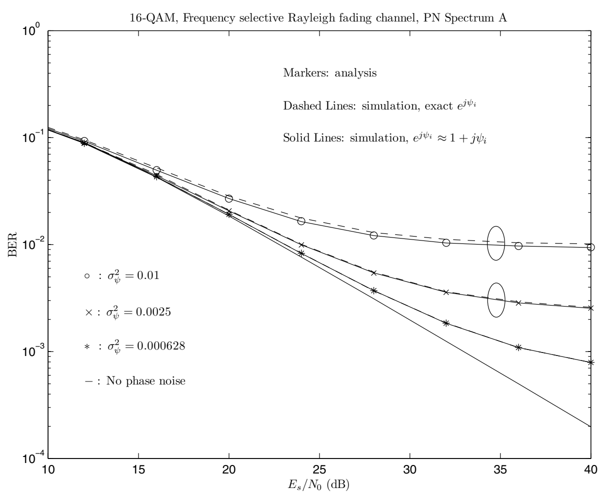

Fig. 3. Effect of phase noise on the BER of 16-QAM modulated OFDM signals in frequency-selective Rayleigh fading channels. Phase noise spectrum A.

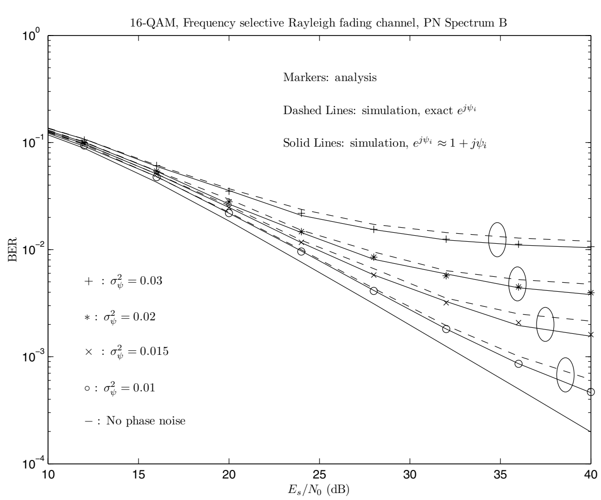

Fig. 4. Effect of phase noise on the BER of 16-QAM modulated OFDM signals in frequency-selective Rayleigh fading channels. Phase noise spectrum B.

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